

# Introduction

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# Introduction

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The contributions in this volume, in their majority, come out of the Colloquium in Honor of Robert Aumann that was held October 11, 2021, at the University Paris–Panthéon–Assas on the occasion of Robert Aumann’s receiving an honoris causa doctorate from the University Paris–Panthéon–Assas. The honoris causa ceremony took place the following day, October 12, in the Grand Amphithéâtre of Sorbonne University.

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Robert John (Yisrael) Aumann, professor emeritus of mathematics at the Hebrew University of Jerusalem, was born in June 1930 in Frankfurt am Main, Germany, to an orthodox Jewish family. Fleeing Nazi persecution, the family emigrated to New York in 1938, where Aumann attended Yeshiva elementary and high school. In 1950, he graduated from the City College of New York, and in 1955, he received his doctorate in mathematics from the Massachusetts Institute of Technology (MIT), with a thesis on knot theory under the supervision of George Whitehead. In 1956, Aumann joined the Mathematics Institute at the Hebrew University of Jerusalem and has remained there to this day. He has held several visiting appointments, most importantly at Princeton, Yale, Berkeley, Stanford, Louvain-la-Neuve, and Stony Brook. Professor Aumann is a member of the American Academy of Arts and Sciences, the National Academy of Sciences (USA), the Israel Academy of Sciences and Humanities, and the British Academy. He holds honorary doctorates from the Universities of Chicago, Bonn, Louvain-la-Neuve, City University of New York, Bar Ilan University, Ben Gurion University, and since 2021 also of the University Paris–Panthéon–Assas. He is a founding member of the Center for the Study of Rationality at the Hebrew University of Jerusalem (1991), the Stony Brook Center for Game Theory, and the Game Theory Society, of which he also served as the first president (1999–2003).

In 2005, Robert Aumann (together with Thomas Schelling) received the Sveriges Riksbank (Bank of Sweden) Prize in Economic Sciences in Memory of Alfred Nobel, for “having enhanced our understanding of conflict and cooperation through game-theory analysis.” In receiving the prize, he was cited for being “the first to conduct a full-fledged formal analysis of so-called infinitely repeated games,” his research having “identified exactly what outcomes can be upheld over time in long-run relations.”

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Robert Aumann is indeed a system builder. He has more than anyone else contributed to creating game theory as a unified field of research in the second half of the 20th century—through his theoretical contributions but also through his teaching and supervising, his many collaborations, and his institutional activities. He has been one of the chief promoters for the transition of game theory from a field known to only a handful of mathematicians and social scientists by the mid of the 20th century to an established field of research with its journals, conferences, professional associations, and institutions. Aumann has worked in both cooperative and non-cooperative game theory, attacking highly abstract—sometimes purely mathematical—as well as applied problems. He is the author of six books and almost 100 research articles. Aumann’s most influential contributions are in the study of repeated games with both complete (Aumann [1959], [1981]; Aumann and Sorin [1989]) and incomplete information (Aumann, Maschler and Stearns [1995]), subjective probability (Anscombe and Aumann [1963]), markets with a continuum of traders (Aumann [1964], [1966], [1975]), bargaining (Aumann and Maschler [1964]), the *value* (Aumann and Shapley [1974]), *correlated equilibrium* (Aumann [1974], [1987]), and the formal definition of *common knowledge* (Aumann [1976]) and interactive epistemology (Aumann [1999a], [1999b]). Aumann’s significance for the field of game theory as a whole can also be read from the fact that his *Collected Papers* have been published when he was only 70 years old, in 2000, five years *before* he received the Nobel Prize. Aumann’s theoretical contributions have a lasting impact, thanks to the particular results that they unearthed but also by force of the conceptual innovations brought along in order to express them. This is distinctively true for his treatment of *repeated games*, the notion of *correlated equilibrium*, and the formal modeling of interactive knowledge, including the set-theoretic notion of *common knowledge* that he provided. In all these areas, Aumann has set up the structure, the “language” in which we think and reason and express our thoughts about these things today.

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## THIS VOLUME

1. Sylvain Sorin’s article, based on his talk at the colloquium, covers Robert Aumann’s contribution to game theory in general. Sorin’s article not only gives an overview of Aumann’s work and assessment of its influence—reviewing his significant contributions and tracing its role in the shaping-up of game theory as a unified field—it is also and foremost a personal tribute to the researcher, the mathematician, and man. “His personality and scientific behavior,” Sorin writes, “have qualified Robert Aumann as the universal attractor for the field of game theory.” Sorin’s article goes beyond a simple survey also in so far as he brings to light structural similarities—in conceptual and mathematical terms—that run “behind” Aumann’s contributions. Written in the form of an essay, Sorin’s contribution represents still and most powerfully a mathematician’s point of view.

2. Françoise Forges in her article, based also on her talk at the colloquium of October 11, 2021, focuses on one of Aumann’s major contributions to the theory of games: *correlated equilibrium*. Aumann introduced the notion of correlated equilibrium as a solution concept for games in strategic form as early as 1974.

Several years later, in 1987, Aumann showed that if players are Bayes rational (that is, if they maximize their individual expected utility with respect to their beliefs) and they share the same prior, then the induced distribution over action profiles is a correlated equilibrium. Several extensions of this notion have been proposed for the framework of games with incomplete information. Forges, one of the early contributors to this research program, gives an account of these extensions in a succinct and authoritative way. In fact, what she offers us is the unique point of view of some kind of backward reading of this literature: taking more recent work by Bergemann and Morris [2013], [2016] and their definition of *Bayes correlated equilibrium* as a starting point, she follows the evolution of the idea backward, over her own account of the problem (Forges [1993]), in which she proposes five different ways of defining correlated equilibrium for games with incomplete information (one of which in terms of communication equilibrium), back to the ideas already implicit in Aumann's work, this way allowing the reader to appreciate the beauty and fecundity of Aumann's original definition of the concept.

3. In his contribution, "Large Economies," presented also at the colloquium of October 11, 2021, Enrico Minelli, for his part, adopts the point of view of economic theory rather than that of game theory. What marks economics as a science is that it attempts to explain the regularities of a social system from individual behavior. In that sense, game theory is very much in line with classical economics. When Robert Aumann, as a young professor of mathematics at Hebrew University on sabbatical at Princeton, came closer into contact with economic theory, the mathematically so far most complete and achieved account of economics as a closed system supported by rationally and decentralized acting individuals—indeed the one outstanding model that served as a benchmark and reference point for all the others—was *general equilibrium theory*. In a series of papers, Aumann [1964], [1966], [1975] established strong connections between solution concepts of cooperative game theory—notably the so-called *core*—and competitive equilibria with a continuum of traders (*large economies*). In doing so, Aumann was the first to have expressed in a mathematically precise way some of the very basic ideas about the formation of prices in markets that can be traced back to Adam Smith and Aristotle. "The bottom line," Aumann explains, looking back at the inspiration for the idea, which came from work by Martin Shubik and Herbert Scarf, "was that, under certain assumptions, the core of a large economy is close to the competitive solution, the solution to which one is led from the law of supply and demand. . . . It [Aumann's theorem] simply says that the core of a large market is the same as the set of competitive outcomes" (Hart [2005], 695).

4. John Geanakoplos and Herakles Polemarchakis in their article "Rational Dialogues," based on Polemarchakis's talk at the colloquium, come back to their dynamic version of Aumann's [1976] "agreeing" theorem. In their 1982 article, "We Can't Disagree Forever," Geanakoplos and Polemarchakis show that if two Bayes rational individuals communicate their posterior of an event back and forth (updating their Bayesian posterior of the event at each step), and if they have the same prior probability distribution over the relevant state space, then they converge to a situation in which their posteriors are common knowledge and—as implied by Aumann's theorem—identical. In their contribution to this volume, they show the following, at first puzzling, result: any sequence of probabilities (no matter what its pattern, with numbers, for instance, going up and down) can be the transcript of a Bayesian dialogue.

5. Lucie Ménager, in “Common Knowledge in Game Theory,” returns to the influence of the formal definition of common knowledge as introduced by Aumann [1976]. In this framework, Aumann shows a result—the already mentioned “agreeing” theorem—that is mathematically as simple as it is conceptually powerful and now famous: if rational agents have the same prior probability and if, at a given state, their posterior probabilities of a given event are common knowledge, then these probabilities have to be identical. This result has given rise to an abundant literature, of which Ménager offers a review here, in particular regarding its implications for the theoretical analysis of exchanges between rational agents and belief convergence, a field of research to which she herself contributed.

6. Lorenzo Bastianello and Vassili Vergopoulos in their article “About Subjective Probability” explore the model of subjective probability as introduced by Anscombe and Aumann [1963]. Bastianello and Vergopoulos summarize the main ideas and the technical framework proposed by Anscombe and Aumann, which has become one of the most widely used in axiomatizations in decision theory, highlighting what they consider Aumann and Anscombe’s main achievements, namely, 1) to “derive subjective probabilities using objective probabilities as a yardstick,” and 2) to propose a framework that allows “the comparison of objective and subjective probabilities.” In this perspective, Bastianello and Vergopoulos propose a novel framework for the derivation of subjective probability based on the Cartesian product of two sets. They show that two simple axioms make it possible to define a subjective probability on a set of states of nature of “primary interest” given an auxiliary set endowed with an objective probability.

7. Michael Greinecker’s article, “Subjectivity and Correlation in Randomized Strategies Revisited,” takes up an issue related to correlated equilibrium. Greinecker shows that many of the results in Aumann’s [1974] original contribution—and importantly also the arguments used to prove them—do not rely on action spaces being finite, which highlights their conceptual nature.

8. Yotam Alexander and Itzhak Gilboa, in “Subjective Causality,” are concerned with the epistemological question of causality—more specifically subjective causation. Under what circumstances do we tend to believe that a previous phenomenon *causes* a later one? Alexander and Gilboa suggest that the subjective perception of causality manifests itself only when the former phenomenon can *explain* the latter in the sense of *reducing its complexity*. They also discuss the relationship of explanation to predictability as sources of the subjective sense of causality in more general set-ups.

9. Alexander Linsbicher in “An Ultra-Refined Grammar for Interactions: Thoughts on Robert Aumann’s Philosophy of Game Theory” explores Aumann’s philosophical and epistemological positions as reflected in Aumann’s explicit engagement with the topic, such as in “What is Game Theory Trying to Accomplish?” (Aumann [1985]) or the interview by Hart [2005].

10. At the colloquium of October 11, 2021, Jörgen Weibull delivered an address to introduce Robert Aumann’s talk. The reprint of his speech also stands here as the overture to Robert Aumann’s own contribution.

11. This volume then closes with Robert Aumann’s contribution, which takes up elements of his talk at the colloquium organized in his honor and his reception speech at the doctorate honoris causa ceremony of the University

Paris–Panthéon–Assas from October 12, 2021. Robert Aumann, at age over 90, as Jörgen Weibull also emphasizes, is still active in research, taking part in debates about recent developments in game and economic theory. For example, in his discussion of behavioral economics (Aumann [2019]), in which he offers a defense of *rule rationality*—which can be seen as a comment and key to his lifework. This is also the underlying theme of Robert Aumann’s own contribution to this volume, in which he looks back to the more fundamental questions that run through his work, defending again the search for abstract regularities as one of the pillars of scientific research and embracing rationality as the most powerful and accurate methodological assumption for studying economic behavior.

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## THANKS

The existence of this special issue of *Revue Économique* in honor of Robert Aumann is in many ways the outcome of a joint effort. It has been made possible by the continuous and generous support of the speakers at the colloquium, including Robert Aumann himself, who agreed to contribute their talks in written form. The authors of the papers collected here have read each other’s works and exchanged comments directly between them in a doubly open fashion. It was a privilege for us, the guest editors of this issue, to have served as the go-between of this process.

Each article speaks in its own voice. And yet what they share is a joint intellectual heritage: the search for order, the attempt to extend the harmony of patterns as it can be found in Aumann’s work; the pleasure of admiring them, sharing them, and making them visible to others, and the wish to pay respect to the mind who has created them.

It is a tremendous pleasure for us to hand this special issue of *Revue Économique* in honor of Robert Aumann over to the public.

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