Reading Barthes

A structural analysis of mathematical writing

Christina Pawlowitsch

Université Panthéon-Assas, Paris II,

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I do not assume that you are a mathematician

(not necessarily so.)

I assume you are a writer . . .

… of essays, research articles, short stories, novels, . . .

Why care?

First defense:

We learn from other forms of expression, composition, creation, art.

. . . I could walk through the gardens and then go to the Musée du Luxembourg where the great paintings were that have now mostly been transferred to the Louvre and the Jeu de Paume. I went there nearly every day for the Cézannes and to see the Manets and the Monets and the other Impressionists that I had first come to know about in the Art Institute at Chicago. I was learning something from the painting of Cézanne that made writing simple true sentences far from enough to make the stories have the dimensions that I was trying to put in them. I was learning very much from him but I was not articulate enough to explain it to anyone. Besides it was a secret.

(Hemingway, Mouvable Feast, 13)

But why?

Why can a writer learn from a painter?

Or a painter from a writer?

Roughly: because both produce signifiers.

(Here starts the Barthesisan accent of this talk.)

Hypothesis: Mathematical writing acquires the form of narrative.

Narrative understood in the very large sense as an abstract sequence of events recounted by a narrator:

There is a narrator and a reader.

There are the actants in the narrative.

It draws on the “language of narrative.”

$\rightarrow$ It has the “structure of narrative.”

And yet mathematical writing it is not narrative fiction.

It is not a novel; not a short story; not a fairy tale.

Narrative does not make see, it does not imitate; the passion that may consume us upon reading a novel is not that of a "vision" (in fact, we "see" nothing), it is the passion of meaning, that is, a higher order of relation, which also carries its emotions, its hopes, its threats, its triumphs: what goes on in a narrative, from the referential (real) point of view, is strictly speaking: *nothing*; "what happens" is language alone, the adventure of language, whose advent never ceases to be celebrated.

(Barthes 1966, Introduction, 26–27)

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Writing mathematics is not that: *it does refer to something* – mathematical objects.

The mathematicians job is to make her reader *come to see those mathematical objects and relations between them*.

Narrative, in Barthes's analysis, is concerned with the bringing into being of sense ("sens"). Storytelling does not refer to anything outside that story. What is brought into being is that story.

On that view:

The story is a signifier – not a thing being signified.

Storytelling is a performative act.

Mathematical writing is not a simple performative act.

There are truth conditions with respect to the things being referred to – which lie beyond of what happens on the level of language – that have to be fulfilled.

Similar to Recanati (1981):

"I declare the session open" is an *illocutionary act in the weak sense*. It will open the session only if the person uttering the phrase is in the position to open the session (if he or she is the chairperson, the president, etc.), that is, if some truth conditions that are beyond the act of uttering the phrase are satisfied.

If these truth conditions are satisfied, then uttering the phrase becomes an *illocutionary act in the strong sense* and will effectively open the session.

Similarly, the difference between mathematics and narrative:

> `Since the graph is closed and since the image of each point under the mapping is convex, we infer from Kakutani's theorem that the mapping has a fixed point (i.e., point contained in its image).  Hence there is an equilibrium point.`

(Nash 1950, Equilibrium points in n-person games, 49)

What makes the above passage a piece of mathematics is that it is true with respect to truth conditions that hinge on the properties of the things referred to (closed graphs and convex mappings).

If these truth conditions were not fulfilled, the two phrases would be some sort of narrative but not a piece of mathematics.

Mathematical notation is like musical notation: it is inanimate as such, but comes to life in the mind of the reader.

What happens is not the experience of reading; it is the experience of coming to work out by oneself – *coming to see* – these mathematical objects and the relations between them.

In this talk: use the terms of investigation proposed by Barthes (1966)

- I. The language of narrative
- II. Functions
- III. Actions
- IV. Narration
- V. The system of narrative

to analyze mathematical writing.

The language of narrative

(La langue du récit)

Beyond the phrase ...

> "The sentence is the smallest segment that perfectly and wholly represents discourse," Barthes quotes Martinet (1961).

Linguistics, Barthes says, stops at the phrase: "the largest unit that it considers to be in its domain of analysis."

Barthes:

Discourse is organized; has its units, rules, and "grammar."
$\rightarrow$ should be the object of a "second linguistics" – a "linguistics of discourse."

> "Discourse is a large sentence just as the sentence is a small discourse."

Subsumes both under "system of sense."

Barthes draws on:

Jakobson and Lévi-Strauss: Humanity can be defined by its capacity to create secondary – multiplicatory – systems: tools to produce other tools, double articulation of language, ...

Ivanov: Artificial languages can be acquired only after natural language; the importance for humanity being to dispose of different "systems of sense."

Barthes:

Postulates a “secondary” – integrative – relation between the sentence and discourse.

The language of narrative is just one of the tongues of the linguistics of discourse

> "A narrative is a big sentence, just as every simple declarative sentence is, in some sense, the sketch of a little narrative."

Barthes suggests three broad types of discourse:

- metonymous (narrative)
- metaphoric (lyrical poetry, wisdom)
- enthymematic (intellectual discursive)

Current study: Second defense

Mathematical writing is enthymematic discourse.

Do we gain some insight by applying the terms of investigation that Barthes suggests for the first type of discourse by applying it to the third?

(And, possibly, does this opposition function as a kind of catalyzer that helps us to shape out more clearly Barthes analysis of narrative in the first place?)

Key concept fo Barthes: different levels of sense – “les niveaux de déscription”

No level can in itself generate sense:

a phoneme makes sense only in combination with the word to which it is attached; a word only within the phrase in which it participates.

Similarly, Barthes says, “a narrative is not a simple sum of propositions.”

> “To understand a narrative is not only to follow the unfolding of the story, but it is also to recognize in it different strata, to project the horizontal sequence of events that builds the thread of the story to an implicitly vertical axis; to read (or to listen) to a story is not only to move from one word to the next, but also from one level to the next.”

“Sense does not sit at the end of a narrative; it runs through it.”

However

Mathematical exposition

- is a simple sequence of propositions.
- Sense does sit a the end of the argument; it *does not* run through it.

Any meaning that possibly arises outside that logical order is *by definition*, outside the domain of mathematics.

Still and all: Mathematicians (can) draw on the language of narrative on the level of the representation

Example: taken from Michael Harris (2012), "Do androids prove theorems in their sleep?"

Robert Thomason and Thomas Trobaugh's (1990) article for the Festschrift of Alexandre Grothendieck's sixtieth birthday.

Why does this work?

Because we are all natives in the language of narrative:

- We have been trained to listen to stories
- Narrative draws on general (universal) linguistic competences
  - integrate one level of sense into a higher level
  - understand communicative implicatures

When we open a novel, we do not really know what there is in it for us ...

Every mathematical text starts with a promise:

I have seen – understood – something, and I want to tell you what it is.

3 levels proposed by Barthes:

- Functions (Propp and Greimas)
- Actions (in the sense that Greimas talks of characters as "actants")
- Narration (what Todorov calls the level of "discours")

Hierarchical order: "a function has sense only in so far as it integrates itself in the general action of an actant; and this action receives its last sense from the fact that it is narrated, entrusted to a discourse that has its own code."

Functions

(Les fonctions)

Is everything in a narrative functional?

Yes, Barthes says:

> "in the domain of discourse, what is noted is, by definition, notable."

> "Even if a detail seems irrefutably insignificant and rebells against each function, it would nonetheless end up pointing out the sense of the absurd or useless."

In mathematical writing, there is no room for the absurd or useless.

Because mathematical writing is reference to something, the functionality of every element can be objectively determined.

But if something is not functional $\rightarrow$ it has to go.

$\rightarrow$ functionality is a necessary condition

Two opposite signs

Narrative fiction		**Mathematical writing**
everything is functional	:	everything is functional
by definition		as a necessary condition

Functional units

Two large classes of functional units:

- *functions* in the strict sense – correlates between acts, as postulated by Tomachevski and Propp, and
- *indices*: properties like personality traits, notations of atmosphere, etc., that do not refer to a complementary or consequential act, but are "nonetheless necessary to the story." Indices are signifiers.

Barthes goes beyond the Russian formalist school – in the very sense of going from one level to the next:

- *functions*
  - *cardinal functions* (nuclei)
  - *catalyzers*: structure time and serve a phatic function: they entertain the contact between the writer and the reader.

"Any notion which in the first place just seems like fulling up the space, always has a discursive function: it precipitates, delays, or revives the discourse, it sums up, anticipates, and sometimes even confuses: what is noted appearing always as being notable, the catalysis constantly reproduces the semantic tension of discourse, does not cease to say: there has been and there will be sense."

- *indices*
  - *indices* properly speaking, and
  - *informants*.

One and the same segment can fulfill multiple functions. For example, a character performing a certain action can have both a function as a catalyzer and as an indicator.

What is not said
– is also functional

"James Bond sees a man of some fifty years."

The implication, Barthes says, is that Bond does not know the man.

Can be analyzed as a *communictive implicature* (Grice) embedded in the narrative

Can mathematical writing exploit this mechanism?

Possibly, on the level of character traits.

Not, on the level of cardinal functions.

But again, the writer should respect it as a constraint that the reader brings this disposition to the text.

Example

When a paper states $P \Rightarrow Q$,

but not that $Q \Rightarrow P$, which would imply that $P \Leftrightarrow Q$,

the expectation is raised that indeed $Q \Rightarrow P$, for other wise the writer would have said so.

But: we can then not say that the author has shown that P does not imply Q and that P and Q are indeed not equivalent.

Actions – characters – actants

(Les actions)

Actors – characters in the narrative

Structural analysis traces character back to *agents* or *actants*; defines characters by their participation in the sphere of actions, and these are classified according to some typology, not based on psychology but on the units of actions assigned to them by the narrative.

Who are the actants in a piece of mathematical writing?

... assumptions, conditions, equations,

propositions, lemmas,

proves,

examples, etc.

Applied mathematics:
narrative within narrative

$\rightarrow$ people or entities in the application

In economics: "consumer" and "firm"

In game theory: "players in the game"

In linguistics: "speaker" and "listener"

Narrator and reader invade the sphere of action of the actants

> `Since the graph is closed and since the image of each point under the mapping is convex, we infer from Kakutani's theorem that the mapping has a fixed point (i.e., point contained in its image). Hence there is an equilibrium point.`

(Nash 1950, Equilibrium points in n-person games, 49)

Observe, however, that (T,R) is strategically unstable: player II knows that player I will never choose B, [..] so if II sees he has to play, he should deduce that player I, who was supposed to play T and was sure to get 2 in this way, certainly did not choose B, where he was sure to get less than 2; player II should thus infer that player I had in fact played M, betting on a chance to get more than 2 (and on the fact that II would understand this signal); and so player II should play L, and hence player I should play M deviating from the equilibrium prescription. [..] We see then that conformity with backwards induction, while being necessary for strategic stability, is not sufficient. (Kohlberg and Mertens 1986, 1007)

In the example:

- All three levels interact
- Two levels of looking on: the narrator makes the reader see the people in the game (the narrative within the narrative) reasoning, and from that the reader and the writer finally “see” something together.

Two opposite signs

Narrative fiction		**Mathematical writing**
the sphere of the narrator and the reader		narrator and reader invade
↑		↓
catalyzers	:	"let," "suppose," "we see"
↑		↓
actants in the narrative invade		the sphere of the actants

Narration

(La narration)

One voice – many voices

Narration usually operates under the convention that there is a unique – grammatically marked – narrative voice.

Not so in mathematical writing:

typically multiplicity of different grammatically marked voices.

THREE CASE STUDIES

- Nash, John F. 1950. Equilibrium points in n-person games.
- Spence, Michael. 1973. Job market signaling.
- Aumann, Robert. 1976. Agreeing to Disagree.

Observations:

- There is a multiplicity of voices or narrative modes: “I” and “we” or different strata of “we” in the same text (“we” as the main narrative voice, “we” as the reader and the writer together, “we” as the scientific community, etc.).

Other observations:

- Mathematical writing takes the form of instructions: callings to imagine or to manipulate objects ("let," "take," "suppose," "add," "substitute"); invitations to "note" or "see" something.
- There is explicit reference to the act of *seeing*: "now we see," "one can see," "it can be seen," "as we have seen," etc.

All three: like the echo of a dialog between the writer and the reader.

Free indirect style

"Free indirect style," Wood (2008) says, "is at its most powerful when hardly visible or audible":

> 'Ted watched the orchestra through stupid tears.' [...] What is so useful about free indirect style is that in our example a word like 'stupid' somehow belongs both to the author and the character; we are not entirely sure who 'owns' the word. [...] Thanks to free indirect style, we see things through the character's eyes and language but also through the author's eyes and language.

Free indirect style – in mathematical papers?

We saw that the "bad" equilibrium 2,2 was sequential; however, it is no longer sequential in the above presentation of the same game ...

(Kohlberg and Mertens 1986, 1008)

The system of narrative

(Le systm̀e du récit)

The *form* of narrative

The narrative *form*, so Barthes, is characterized by two fundamental operations:

- to stretch out its signs throughout the story, and
- the possibility to insert into these threads of stretched out signs (unpredictable) expansions.

> *... these two forces appear as liberties; unique to narrative, however, is precisely that it includes in its language these "deviations."*

Example:

In real life, the invitation to sit down is immediately followed by the act of sitting down; not so in narrative: a whole sequence that belongs to some other sphere of function can be interjected.

→ narrative constitutes its own "logic time" – le temps logique du récit

The catalyzer has as its corollary the ellipsis:

it is possible to reduce a sequence to its kernels and a hierarchy of sequences can be summed-up by its highest terms without changing the sens of the story.

Narrative can be summarized:

it is possible to extract the *argument* of a narrative

$\rightarrow$ mathematical writing is pure argument.

Sense

What is separated on one level is collected on a higher level

Often, the same sequence can be reabsorbed into higher orders of different degrees

$\rightarrow$ in mathematical writing: very often the case

The “realism” of narrative has to be fended off, Barthes says:

> *… the function of narrative is not to “represent,” it is to set up a performance that remains enigmatic to us and that cannot be in the order of mimicry; the “reality” of a sequence is not in the “natural” sequence of actions of which it is composed, but in the logic that it exhibits, to which it exposes itself, that it satisfies.*

And Barthes closes:

> *Narrative does not make see, it does not imitate; the passion that may consume us upon reading a novel is not that of a "vision" (in fact, we "see" nothing), it is the passion of meaning, that is, a higher order of relation, which also carries its emotions, its hopes, its threats, its triumphs: what goes on in a narrative, from the referential (real) point of view, is strictly speaking:* nothing; *"what happens" is language alone, the adventure of language, whose advent never ceases to be celebrated.*

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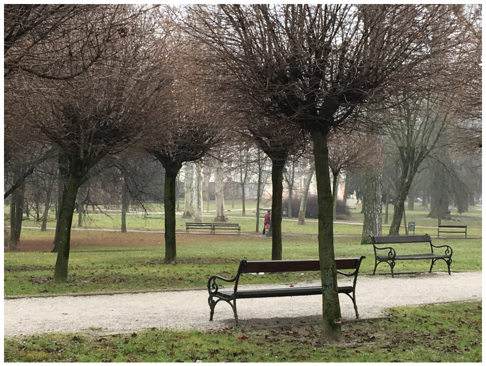

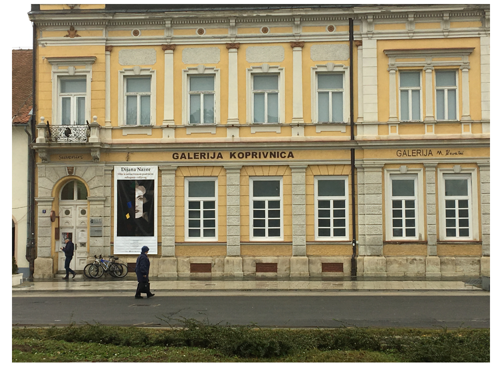

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