Costly Signaling: Rationality and Evolution

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Spence (1973) : *Signaling in the job market*

The term "market signaling" is not exactly a part of the well-defined, technical vocabulary of the economist. As a part of the preamble, therefore, I feel I owe the reader a word of explanation about the title. I find it difficult, however, to give a coherent and comprehensive explanation of the meaning of the term abstracted from the contents of the essay. In fact, it is part of my purpose to outline a model in which signaling is implicitly defined and to explain why one can, and perhaps should, be interested in it.

Spence (1973) : *Signaling in the job market*

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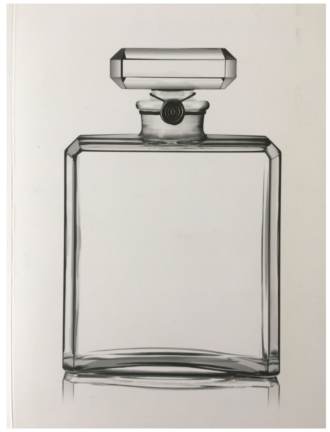
Costly-signaling games – wide range of applications



- Veblen (1899): Theory of "conspicuous consumption"
- Spence (1973): Signaling in the job market: education as a costly signal
- Zahavi (1975), Grafen (1990): "The Handicap Principle": grounding Darwin's theory of sexual selection through mate choice in natural selection

$\downarrow$

- Dawkins and Krebs (1978): emphasize the possibility of "cheating"
- Miller and Rock (1985): dividend policy as a costly signal
- Milgrom and Roberts (1986): advertising as a costly signal
- Caro (1986): signals in predator–prey interaction ("stotting")
- Bliege Bird and Smith (2005): inefficient foraging strategies, gift-giving, communal sharing, rituals, embodied handicaps as costly signals
- Van Rooy (2003): politeness in language as a costly signal
- Archetti (2008): signals in parasite–host interaction (color of autumn leaves)

Veblen (1899): *The Theory of the Leisure Class*:
estates, dress, education, taste in art as "conspicuous consumption"

Zahavi (1975): "The Handicap Principle": grounding Darwin's theory of sexual selection through mate choice in natural selection

Dawkins and Krebs (1978): emphasize the possibility of "cheating"

Grafen (1990): formal model

Caro (1986): signals in predator–prey interaction ("stotting")

Archetti (2008): signals in parasite–host interaction (color of autumn leaves)

Forbes

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In Low-Yield World, Global Investors Turn To Big Dividend Foreign Stocks

Kenneth Rapoza Senior Contributor

Markets

I write about business and investing in emerging markets.

Miller and Rock (1985): dividend policy as a costly signal

Milgrom and Roberts (1986): advertising as a costly signal

Bliege Bird and Smith (2005): inefficient foraging strategies, gift-giving, communal sharing, rituals, embodied handicaps as costly signals

Van Rooy (2003): politeness in language as a costly signal (informal argument)

Pawlowitsch and Hofbauer (2019, book manuscript): politeness, accents + formal model

This work: threefold "binary" costly-signaling games: 2 states of nature, 2 signals, 2 actions

Five classes

(I) production of a costly signal is of different costs for different types — discrete version of Spence's (1973) model

(II) production of a costly signal is of the same cost for different types, but types have different benefits if the signal has the desired effect — discrete version of Milgrom and Robert's (1986) model of advertising and Grafen's (1990) formalization of the handicap principle

(III) production of a costly signal is of the same cost for different types and types have a different background payoff throughout

(IV) different signals are costly for different types

(V) different signals generate a positive payoff increment for different types, similar Cho and Kreps's (1986) *beer-quiche game*

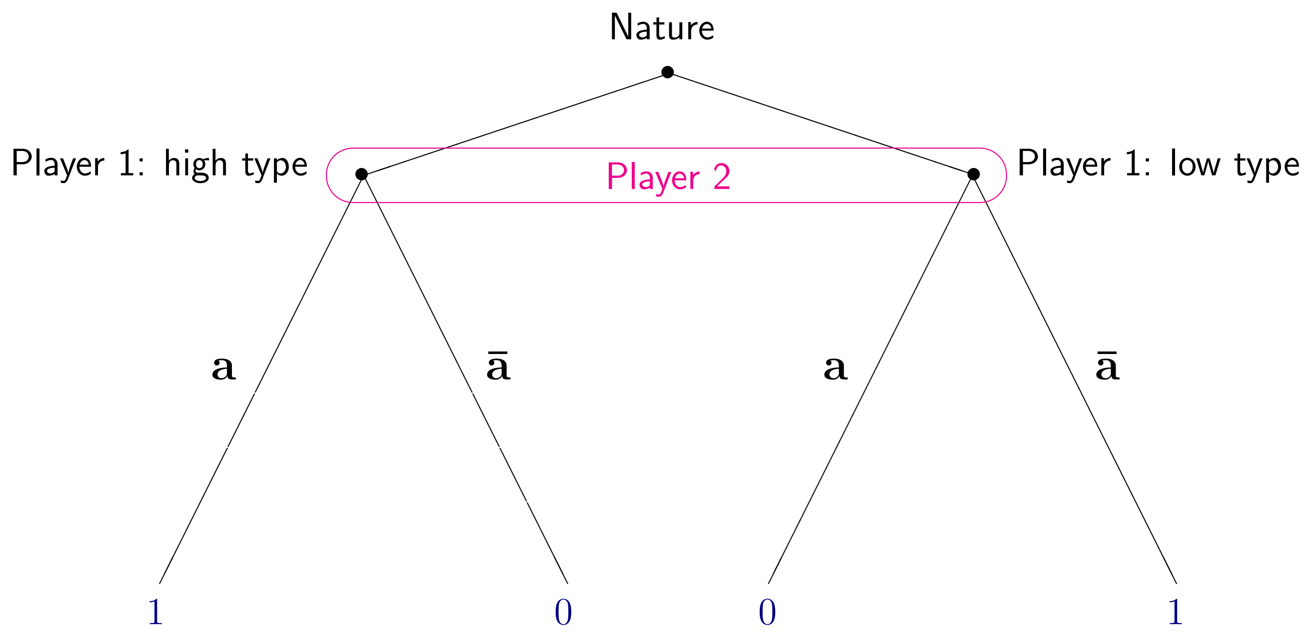
- complete equilibrium structure
- stability of equilibria:
  - ⋆ *Strategic stability*: rationality-oriented, refinements of Nash equilibrium
    → we build on Govindan and Wilson (2009)
  - ⋆ *Index theory*
  - ⋆ *Evolutionary stability*:
    → we complement previous work by Cressman (2003), Wagner (2013), Zollman et al. (2013)

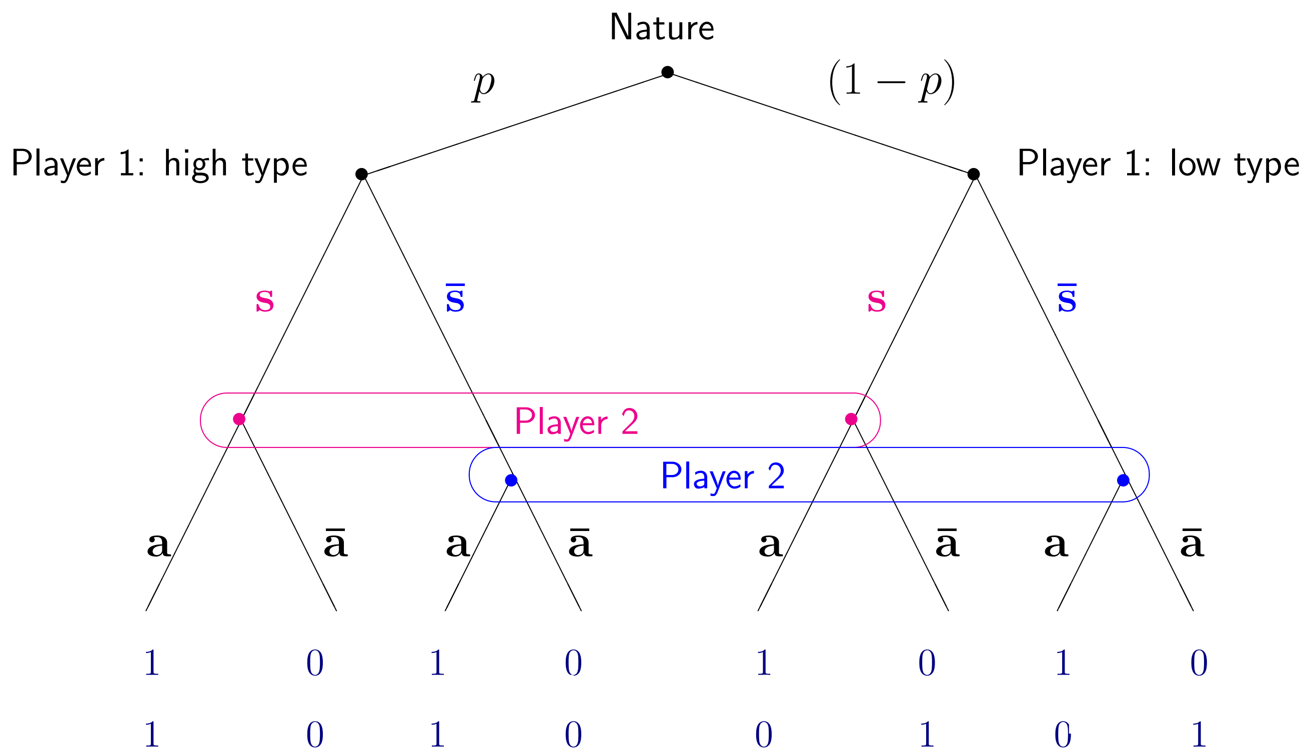
Costly-signaling theory

begins with

a problem of cooperation

– under uncertainty


Nature
Player 1: high type
Player 2
Player 1: low type
a
ā
a
ā
1
0
0
1


Nature
p
(1 − p)
Player 1: high type
Player 1: low type
s
s̄
s
s̄
Player 2
Player 2
a
ā
a
ā
a
ā
a
ā
1
0
1
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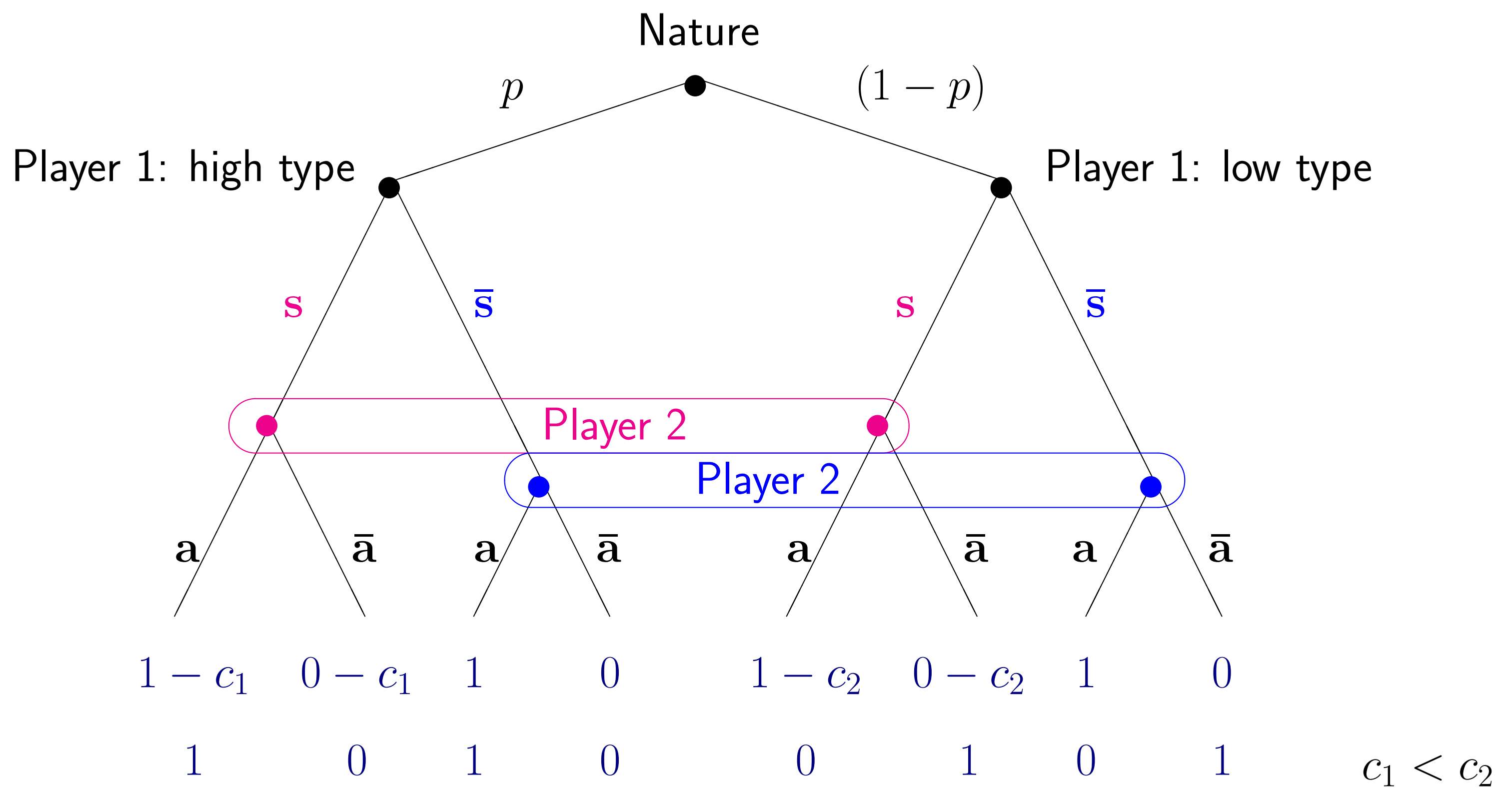
1
0
1

Class I: different costs of the signal – same background payoff

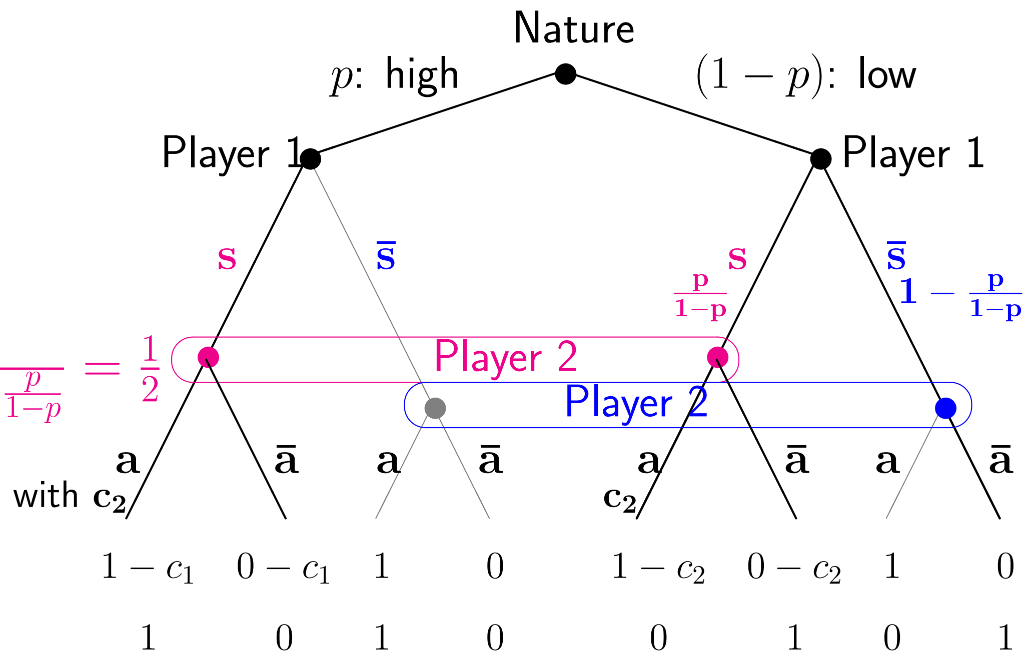


	$\mathbf{aa}$	$\mathbf{a\bar{a}}$	$\mathbf{\bar{a}a}$	$\mathbf{\bar{a}\bar{a}}$
$\mathbf{ss}$	$1-pc_1-(1-p)c_2,\ p$	$1-pc_1-(1-p)c_2,\ p$	$-pc_1-(1-p)c_2,\ 1-p$	$-pc_1-(1-p)c_2,\ 1-p$
$\mathbf{s\bar{s}}$	$1-pc_1,\ p$	$p(1-c_1),\ 1$	$-pc_1+(1-p),\ 0$	$-pc_1,\ 1-p$
$\mathbf{\bar{s}s}$	$1-(1-p)c_2,\ p$	$(1-p)(1-c_2),\ 0$	$p-(1-p)c_2,\ 1$	$-(1-p)c_2,\ 1-p$
$\mathbf{\bar{s}\bar{s}}$	$1,\ p$	$0,\ 1-p$	$1,\ p$	$0,\ 1-p$

Class I, $0 < c_1 < c_2 < 1$: $p < \frac{1}{2}$: partially revealing equilibrium

Bayes' law:

$$p(h \mid s) = \frac{p}{p+(1-p)\cdot\frac{p}{1-p}} = \frac{1}{2}$$



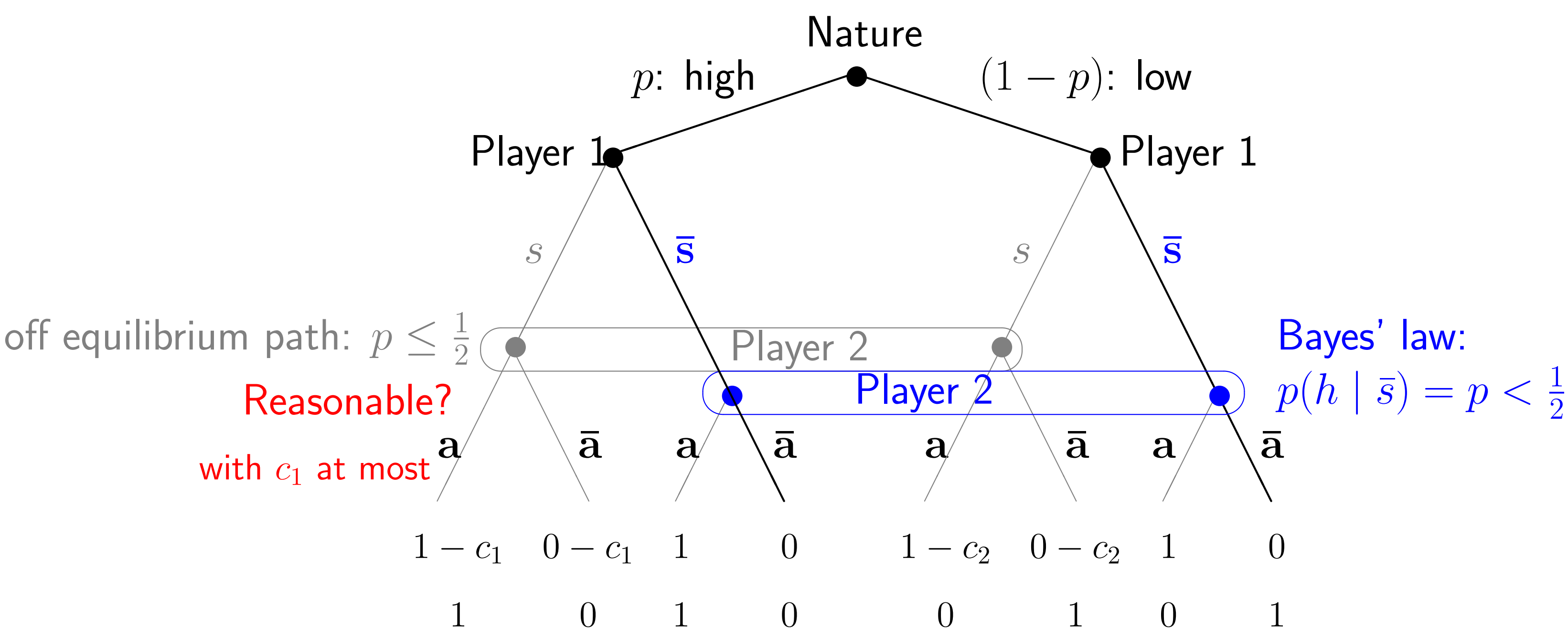
Bayes' law:

$$p(h \mid \bar{s}) = 0$$

		$\mathbf{aa}$	$\mathbf{c_2 \to a\bar{a}}$	$\mathbf{\bar{a}a}$	$\mathbf{1-c_2 \to \bar{a}\bar{a}}$
$\frac{\mathbf{p}}{\mathbf{1-p}} \to$	$\mathbf{ss}$	$1-pc_1-(1-p)c_2,\ p$	$\mathbf{1-pc_1-(1-p)c_2},\ p$	$-pc_1-(1-p)c_2,\ \mathbf{1-p}$	$-pc_1-(1-p)c_2,\ \mathbf{1-p}$
$\to$	$\mathbf{s\bar{s}}$	$1-pc_1,\ p$	$p(1-c_1),\ \mathbf{1}$	$-pc_1+(1-p),\ 0$	$-pc_1,\ 1-p$
	$\mathbf{\bar{s}s}$	$1-(1-p)c_2,\ p$	$(1-p)(1-c_2),\ 0$	$p-(1-p)c_2,\ \mathbf{1}$	$-(1-p)c_2,\ 1-p$
	$\mathbf{\bar{s}\bar{s}}$	$\mathbf{1},\ p$	$0,\ \mathbf{1-p}$	$\mathbf{1},\ p$	$\mathbf{0},\ \mathbf{1-p}$

- E1: Partially revealing: 1 between ss and $s\bar{s}$ with $\frac{p}{1-p}$; 2 between $a\bar{a}$ and $\bar{a}\bar{a}$, with c_2.

Class I, $0 < c_1 < c_2 < 1$, $p < \frac{1}{2}$: "no-signaling" equilibrium outcome



	$\mathbf{aa}$	with $y \in [0, c_1] \to \mathbf{a\bar{a}}$	$\mathbf{\bar{a}a}$	with $1-y \to \mathbf{\bar{a}\bar{a}}$
$\mathbf{ss}$	$1-pc_1-(1-p)c_2,\ p$	$\mathbf{1-pc_1-(1-p)c_2},\ p$	$-pc_1-(1-p)c_2,\ \mathbf{1-p}$	$-pc_1-(1-p)c_2,\ \mathbf{1-p}$
$\mathbf{s\bar{s}}$	$1-pc_1,\ p$	$p(1-c_1),\ \mathbf{1}$	$-pc_1+(1-p),\ 0$	$-pc_1,\ 1-p$
$\mathbf{\bar{s}s}$	$1-(1-p)c_2,\ p$	$(1-p)(1-c_2),\ 0$	$p-(1-p)c_2,\ \mathbf{1}$	$-(1-p)c_2,\ 1-p$
$\mathbf{\bar{s}\bar{s}}$	$\mathbf{1},\ p$	$0,\ \mathbf{1-p}$	$\mathbf{1},\ p$	$\mathbf{0},\ \mathbf{1-p}$

- P1: No-signaling: 1 takes $\bar{s}\bar{s}$; 2 mix between $a\bar{a}$ and $\bar{a}\bar{a}$ with $y \in [0, c_1]$ on first.

Equilibrium structure

$p < \frac{1}{2}$:	(E1) : *partially revealing*	$\mathbf{h} \longrightarrow \mathbf{s}$	$\mathbf{s} \longrightarrow p^\star = \frac{1}{2}$: $\mathbf{a}$ with c_2
		$\mathbf{l} \longrightarrow \mathbf{s}$ with $\frac{\mathbf{p}}{\mathbf{1-p}}$	$\bar{\mathbf{s}} \longrightarrow$ low for sure : $\bar{\mathbf{a}}$
	(P1): *both use $\bar{s}$*	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$	$\mathbf{s} \longrightarrow a$ with prob $\leq c_1$
		$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow p^\star = p < \frac{1}{2}$: $\bar{\mathbf{a}}$
$p > \frac{1}{2}$:	(E2) : *partially revealing*	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$ with $\frac{\mathbf{1-p}}{\mathbf{p}}$	$\mathbf{s} \longrightarrow$ high for sure : $\mathbf{a}$
		$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow p^\star = \frac{1}{2}$: $\mathbf{a}$ with $1 - c_1$
	(P2): *both use s*	$\mathbf{h} \longrightarrow \mathbf{s}$	$\mathbf{s} \longrightarrow p^\star = p > \frac{1}{2}$: $\mathbf{a}$
		$\mathbf{l} \longrightarrow \mathbf{s}$	$\bar{\mathbf{s}} \longrightarrow \mathbf{a}$ with prob $\leq 1 - c_2$
	(P3): *both use $\bar{s}$*	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$	$\mathbf{s} \longrightarrow \mathbf{a}$ with any prob
		$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow p^\star = p > \frac{1}{2}$: $\mathbf{a}$
$p = \frac{1}{2}$:	(E1-P2): *both use s*	$\mathbf{h} \longrightarrow \mathbf{s}$	$\mathbf{s} \longrightarrow p^\star = p = \frac{1}{2}$: $\mathbf{a}$ with $y \in [c_2, 1]$
		$\mathbf{l} \longrightarrow \mathbf{s}$	$\bar{\mathbf{s}} \longrightarrow \mathbf{a}$ with $y' \in [0, y - c_2]$
	(P1-E2-P3): *both use $\bar{s}$*	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$	$\mathbf{s} \longrightarrow \mathbf{a}$ with $y \in [0, \min\{y' + c_1, 1\}]$
		$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow p^\star = p = \frac{1}{2}$: $\mathbf{a}$ with $y' \in [0, 1]$

Equilibrium selection? How to refine the equilibrium notion?

- In classical game theory:

  $\longrightarrow$ restrictions on beliefs "off the equilibrium path" ($=$ after an unused signal)

  - Banks and Sobel (1987): "divinity"
  - Govindan and Wilson (2009): "forward induction"

    $\rightarrow$ coincide here and discard no-signaling equilibrium outcome P1

  - Cho and Kreps (1987): "intuitive criterion" has no selection force here (except in case $p = 1/2$)

“Forward-induction” argument

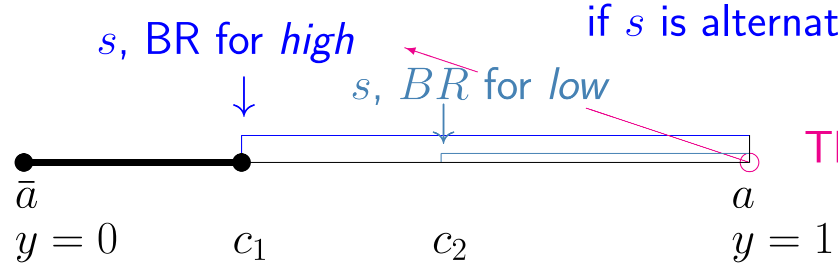
P1: both types $\bar{s}$; 2 in response to $\bar{s}$ takes $\bar{a}$;
off equilibrium path: in response to s, takes a with c_1 at most

$\longrightarrow$ implies that after s, player 2 attributes to the high type a belief of $\frac{1}{2}$ at most! Not “plausible” (by various criteria) $\rightarrow$ equilibrium discarded

“Never-a-weak BR” and “Divinity”

After unused signal s:

Forward induction (Govindan and Wilson 2009)
$\Leftrightarrow$ “never-a-weak BR”:
after s, type maintained only
if s is alternative BR for him $\Rightarrow$ only *high* survives

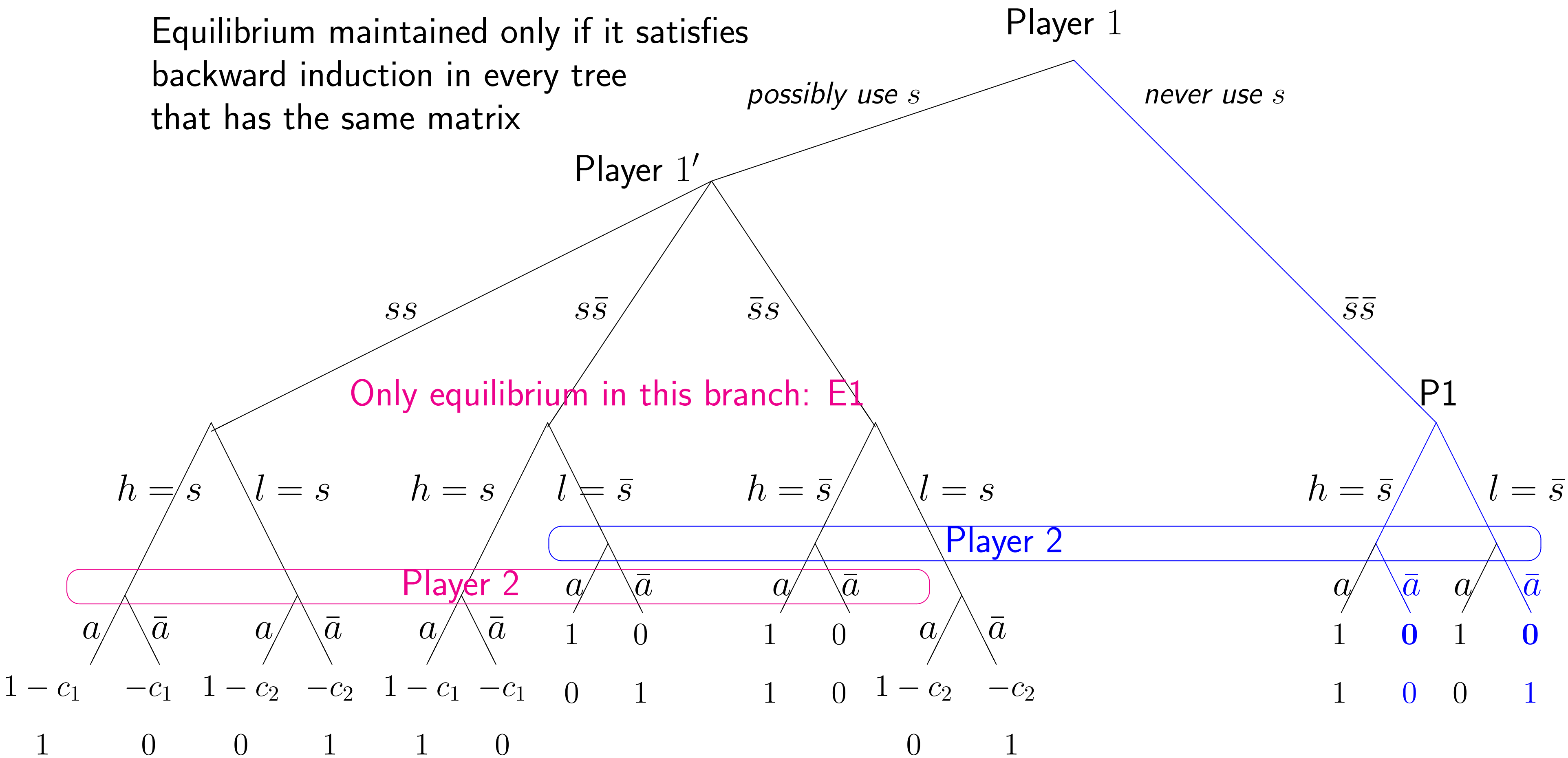


Then, player 2, when s should take a
$\Rightarrow$ clash with equilibrium P1!

Divinity (Banks and Sobel 1987):
after s, type maintained only
if there is no other type who has a larger better-off set

Forward induction (Govindan and Wilson 2009): foundation in "invariance + sequentiality"

Equilibrium maintained only if it satisfies
backward induction in every tree
that has the same matrix



This tree has the same matrix as class I. But P1 (both use $\bar{s}$) not backward induction! $\longrightarrow$ E1 only backward-induction equilibrium!

The index: a rough guide to evolutionary stability

Shapley (1974): Index, $+1$ or -1, to every regular equilibrium

- Strict equilibrium has index $+1$.
- Removing or adding unused strategies does not change the index.
- *Index Theorem*: the sum of the indices of all equilibria is $+1$.

Hofbauer and Sigmund (1988, 1998): index as the sign of the determinant of the negative Jacobian

Ritzberger (1994, 2002): index of an equilibrium component is:

- an integer
- robust under payoff perturbations

Demichelis and Ritzberger (2003):

- If an equilibrium component is asymptotically stable under some evolutionary dynamics, then its index equals its Euler characteristics.
- Special case: If it is convex or contractible, then its index is $+1$.

In our game:

$p < 1/2$:

- E1: Isolated and quasistrict $\longrightarrow$ regular
  - removing unused strategies $\longrightarrow$ 2×2 cyclic game
  - in this game, E1 only equilibrium $\longrightarrow$ index $+1$

  $\Rightarrow$ candidate for asymptotically stable equilibrium
- P1: by Index Theorem $\longrightarrow$ index 0

  $\Rightarrow$ not asymptotically stable, under no evolutionary dynamics

$p > 1/2$:

- P2: by robustness $\longrightarrow$ index $+1$
- E2: Isolated and quasistrict $\longrightarrow$ regular
  - removing unused strategies $\longrightarrow$ 2×2 coordination game with 3 equilibria: E2 and two strict equilibria (index $+1$)
  - by Index Theorem $\longrightarrow$ index -1.
- P3: by Index Theorem $\longrightarrow$ index $+1$

Equilibrium structure

$p < \frac{1}{2}$:	(E1) : *partially revealing*	$\mathbf{h} \longrightarrow \mathbf{s}$	$\mathbf{s} \longrightarrow p^\star = \frac{1}{2}$: $\mathbf{a}$ with c_2
	Index: $+1$. Fwd Ind	$\mathbf{l} \longrightarrow \mathbf{s}$ with $\frac{\mathbf{p}}{\mathbf{1-p}}$	$\bar{\mathbf{s}} \longrightarrow$ low for sure : $\bar{\mathbf{a}}$
	(P1): *both use* $\bar{s}$	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$	$\mathbf{s} \longrightarrow a$ with prob $\leq c_1$
	Index: 0. Not Fwd Ind	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow p^\star = p < \frac{1}{2}$: $\bar{\mathbf{a}}$
$p > \frac{1}{2}$:	(E2) : *partially revealing*	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$ with $\frac{\mathbf{1-p}}{\mathbf{p}}$	$\mathbf{s} \longrightarrow$ high for sure : $\mathbf{a}$
	Index: -1. Fwd Ind	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow p^\star = \frac{1}{2}$: $\mathbf{a}$ with $1 - c_1$
	(P2): *both use* s	$\mathbf{h} \longrightarrow \mathbf{s}$	$\mathbf{s} \longrightarrow p^\star = p > \frac{1}{2}$: $\mathbf{a}$
	Index: $+1$. Fwd Ind	$\mathbf{l} \longrightarrow \mathbf{s}$	$\bar{\mathbf{s}} \longrightarrow \mathbf{a}$ with prob $\leq 1 - c_2$
	(P3): *both use* $\bar{s}$	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$	$\mathbf{s} \longrightarrow \mathbf{a}$ with any prob
	Index: $+1$. Fwd Ind	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow p^\star = p > \frac{1}{2}$: $\mathbf{a}$
$p = \frac{1}{2}$:	(E1-P2): *both use* s	$\mathbf{h} \longrightarrow \mathbf{s}$	$\mathbf{s} \longrightarrow p^\star = p = \frac{1}{2}$: $\mathbf{a}$ with $y \in [c_2, 1]$
	Index: $+1$ Fwd Ind	$\mathbf{l} \longrightarrow \mathbf{s}$	$\bar{\mathbf{s}} \longrightarrow \mathbf{a}$ with $y' \in [0, y - c_2]$
	(P1-E2-P3): *both use* $\bar{s}$	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$	$\mathbf{s} \longrightarrow \mathbf{a}$ with $y \in [0, \min\{y' + c_1, 1\}]$
	Index: 0. Not Fwd Ind	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow p^\star = p = \frac{1}{2}$: $\mathbf{a}$ with $y' \in [0, 1]$

Evolutionary Dynamics

Payoffs

$$\begin{aligned} u^1(ss, \mathbf{y}) &= y - pc_1 - (1-p)c_2 \\ u^1(s\bar{s}, \mathbf{y}) &= p(y - c_1) + (1-p)y' \\ u^1(\bar{s}s, \mathbf{y}) &= (1-p)(y - c_2) + py' \\ u^1(\bar{s}\bar{s}, \mathbf{y}) &= y' \end{aligned} \tag{1}$$

$\mathbf{y} = (y(aa), y(a\bar{a}), y(\bar{a}a), y(\bar{a}\bar{a}))$, a mixed strategy of player 2
$y = y(aa) + y(a\bar{a})$
$y' = y(aa) + y(\bar{a}a)$

Note that:

$$u^1(ss) + u^1(\bar{s}\bar{s}) = u^1(s\bar{s}) + u^1(\bar{s}s) \tag{2}$$

Similarly:

$$
\begin{aligned}
u^2(aa, \mathbf{x}) &= p \\
u^2(a\bar{a}, \mathbf{x}) &= px_h + (1-p)(1-x_\ell) \\
u^2(\bar{a}a, \mathbf{x}) &= p(1-x_h) + (1-p)x_\ell \\
u^2(\bar{a}\bar{a}, \mathbf{x}) &= 1-p
\end{aligned}
\tag{3}
$$

$\mathbf{x} = (x(ss), x(s\bar{s}), x(\bar{s}s), x(\bar{s}\bar{s}))$,
$x_h = x(ss) + x(s\bar{s})$,
$x_\ell = x(ss) + x(\bar{s}s)$

Note that also:

$$u^2(aa) + u^2(\bar{a}\bar{a}) = 1 = u^2(a\bar{a}) + u^2(\bar{a}a) \tag{4}$$

Replicator dynamics for the normal form game

$$\dot{x}_i = x_i(u_i^1 - \overline{u^1}), \quad \dot{y}_j = y_j(u_j^2 - \overline{u^2}) \qquad \text{(REP)}$$

special feature: $u_1 + u_4 = u_2 + u_3$ (for both players)

Gaunersdorfer, Hofbauer, and Sigmund (1991):

in that case, $\frac{x_1 x_4}{x_2 x_3}$ and $\frac{y_1 y_4}{y_2 y_3}$ are constants of motion for (REP)
$\to$ foliation of state space $\Delta_4 \times \Delta_4$ into 4d invariant manifolds

The 'central' invariant manifold $x_1 x_4 = x_2 x_3$ (the Wright manifold) can be parameterized by
$x_1 = xx', x_2 = x(1 - x'), x_3 = (1 - x)x', x_4 = (1 - x)(1 - x')$
with $(x, x') \in [0, 1]^2$: $x = x_1 + x_2, x' = x_1 + x_3$.

On this invariant manifold, (REP) can be written as

$$\begin{aligned} \dot{x} &= x(1 - x)(u_1 - u_3) \\ \dot{x}' &= x'(1 - x')(u_1 - u_2) \end{aligned} \qquad (5)$$

In our game:

On the ‘central’ invariant manifold (the Wright manifold)

$$x(ss)x(\bar{s}\bar{s}) = x(s\bar{s})x(\bar{s}s), \quad y(aa)y(\bar{a}\bar{a}) = y(a\bar{a})y(\bar{a}a)$$

with $x_h = x(ss) + x(s\bar{s})$, $x_\ell = x(ss) + x(\bar{s}s)$
and $y = y(aa) + y(a\bar{a}), y' = y(aa) + y(\bar{a}a)$:

$$\begin{aligned}
\dot{x}_h &= x_h(1 - x_h)(y - c_1 - y')p \\
\dot{x}_\ell &= x_\ell(1 - x_\ell)[y - c_2 - y'](1 - p) \\
\dot{y} &= y(1 - y)[px_h - (1 - p)x_\ell] \\
\dot{y}' &= y'(1 - y')[p(1 - x_h) - (1 - p)(1 - x_\ell)]
\end{aligned} \tag{6}$$

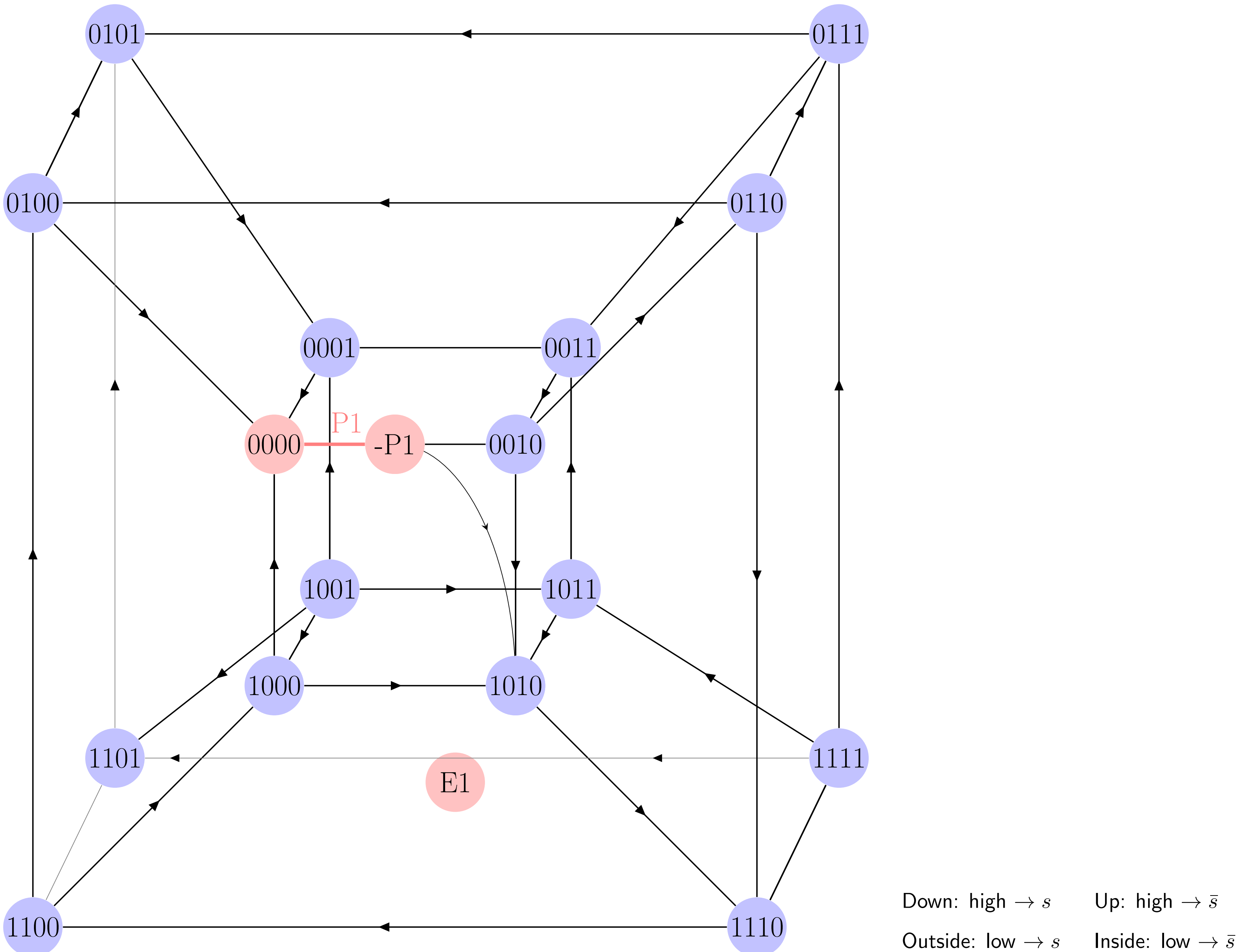
Replicator dynamics for behavior strategies

Interpret:

$x_h = \text{prob}(s|\text{high})$, $x_\ell = \text{prob}(s|\text{low})$,
$y = \text{prob}(a|s)$, $y' = \text{prob}(a|\bar{s})$.

(6) looks like the replicator equation for a binary 4 person game with linear incentives.

State space: (x_h, x_ℓ, y, y') in hypercube $[0, 1]^4$


0101
0111
0100
0110
0001
0011
P1
0000
-P1
0010
1001
1011
1000
1010
1101
1111
E1
1100
1110
Down: high $\to s$
Up: high $\to \bar{s}$
Outside: low $\to s$
Inside: low $\to \bar{s}$
Left: after $s \to \bar{a}$
Right: after $s \to a$
Front: after $\bar{s} \to \bar{a}$
Back: after $\bar{s} \to a$

Analysis of dynamics:

$p < \frac{1}{2}$: Rest points: all 2^4 corners of the hypercube as well as E1 and the edges $(0,0,*,0), (0,0,*,1), (1,1,0,*), (1,1,1,*)$.

Dynamics near the partially revealing E1 $= (1, \frac{p}{1-p}, c_2, 0)$: E1 is a quasistrict Nash equilibrium: external eigenvalues:

$$\frac{(1-x_h)^{\cdot}}{1-x_h} = (c_2 - c_1)p < 0, \quad \frac{\dot{y}'}{y'} = 2p - 1 < 0$$

In the supporting boundary face $x_h = 1, y' = 0$:

$$\begin{aligned} \dot{x}_\ell &= x_\ell(1-x_\ell)[y - c_2](1-p) \\ \dot{y} &= y(1-y)[p - (1-p)x_\ell] \end{aligned} \tag{7}$$

replicator dynamics for a cyclic 2×2 game, with closed orbits around the equilibrium E1. Each of these periodic solutions: two external eigenvalues (Floquet exponents) that equal the two external eigenvalues at E1 (by the averaging property of replicator dynamics) $\rightarrow$ attracts a 3-dimensional manifold of solutions. Boundary face $x_h = 1, y' = 0$ attracts an open set of initial conditions from $[0,1]^4$.

Dynamics near the edge containing P1, $(0,0,y,0)$:

Near $(0,0,y,0)$, the linearized dynamics:

$$\begin{aligned} \dot{x}_h/x_h &= (y-c_1)p \\ \dot{x}_\ell/x_\ell &= (y-c_2)(1-p) \\ \dot{y}'/y' &= p-(1-p)<0 \end{aligned} \tag{8}$$

so these are NE for $0 \leq y \leq c_1$ (component P1). For $0 \leq y < c_1$, all three external eigenvalues are negative, hence this is a quasistrict NE and attracts a 3d stable manifold. The basin of attraction of the whole component P1 contains an open set from the hypercube.

The end point of the component P1, -P1$=(0,0,c_1,0)$ is unstable. This point has a 2d stable manifold and a 2d center manifold, the latter contained in the 2d face $x_\ell = y' = 0$ with dynamics

$$\begin{aligned} \dot{x}_h &= x_h(1-x_h)[y-c_1]p \\ \dot{y} &= y(1-y)px_h \end{aligned} \tag{9}$$

the replicator dynamics of a degenerate/nongeneric 2×2 game. There is one orbit converging to -P1, and one orbit with -P1 as α–limit which converges to the corner $(1,0,1,0)$

Convergence

$$
\begin{aligned}
\dot{x}_h &= x_h(1-x_h)(y-c_1-y')p \\
\dot{x}_\ell &= x_\ell(1-x_\ell)[y-c_2-y'](1-p) \\
\dot{y} &= y(1-y)[px_h-(1-p)x_\ell] \\
\dot{y}' &= y'(1-y')[p(1-x_h)-(1-p)(1-x_\ell)]
\end{aligned}
\tag{10}
$$

We show that all orbits in the interior of the hypercube converge to either the face spanned by E1 or to the component P1. On the boundary, orbits may also converge to one of the other rest points.

From the first two equations of (10) we see that

$$
\frac{\dot{x}_h}{px_h(1-x_h)} - \frac{\dot{x}_\ell}{(1-p)x_\ell(1-x_\ell)} = c_2 - c_1 > 0 \tag{11}
$$

and hence

$$
\frac{1}{p}[\log x_h - \log(1-x_h)]^{\cdot} - \frac{1}{1-p}[\log x_\ell - \log(1-x_\ell)]^{\cdot} = c_2 - c_1 > 0
$$

and

$$
\left[\frac{x_h}{1-x_h}\right]^{1-p}\left[\frac{1-x_\ell}{x_\ell}\right]^{p} \uparrow \infty
$$

Since the numerators are bounded we infer

$$(1 - x_h)x_\ell \to 0 \tag{12}$$

so that all interior orbits converge to the union of the two faces $x_\ell = 0$ and $x_h = 1$.

Similarly, we obtain from the last two equations of (10)

$$[\log y - \log(1-y) + \log y' - \log(1-y')]^{\cdot} = \frac{\dot{y}}{y(1-y)} + \frac{\dot{y}'}{y'(1-y')} = 2p - 1 < 0 \tag{13}$$

and hence

$$yy' \to 0$$

(since $p < \frac{1}{2}$) so that all interior orbits converge to the union of the two faces $y = 0$ and $y' = 0$. Together the ω–limit sets must be contained in the union of four 2d faces:

$(1, *, 0, *)$ (there all orbits converge to $(1, 0, 0, 0)$),

$(1, *, *, 0)$ (this is the face containing E1 and the periodic solutions),

$(*, 0, 0, *)$ (there all orbits converge to $(0, 0, 0, 0)$), and

$(*, 0, *, 0)$ (the dynamics on this face—which contains the equilibrium component P1 in an edge—was described above).

Best-reponse dynamics

All orbits converge to one of the NE: either to E1, or to P1. This follows e.g. from Berger (2005), since we can reduce the 4×4 game to a 3×2 game (for $p < \frac{1}{2}$).

E1 is asymptotically stable, the component P1 is not.

Both components attract big open sets. Most orbits converging to P1 converge to the corner $(0, 0, 0, 0)$.

$\mathbf{p > \frac{1}{2}}$:

Here (REP) has the following rest points: all 2^4 corners of the hypercube, the edges $(1,1,0,*)$ and $(1,1,1,*)$ where player 1 always sends the signal, the latter contains the NE component P2, the edges $(0,0,*,0)$ and $(0,0,*,1)$ where player 1 never sends the signal, the latter is the NE component P3, and—instead of E1 (which is now outside the hypercube)—there is an isolated Nash equilibrium at E2 $= (1 - \frac{1-p}{p}, 0, 1, 1 - c_1)$.

The expression in (13) is now positive for $p > \frac{1}{2}$, and hence

$$(1-y)(1-y') \to 0.$$

This means that all interior orbits converge to the union of the two facets $y = 1$ and $y' = 1$. Together with (12) (which holds for all $p \in (0,1)$) the ω–limit sets must be contained in the union of four 2d faces:

$(1, *, 1, *)$ — this face contains the edge of restpoints $(1, 1, 1, *)$; interior orbits in this face converge to one of the NE $(1, 1, 1, y')$ with $0 < y' < 1 - c_2$.

$(1, *, *, 1)$ — this face contains again the edge of restpoints $(1, 1, 1, *)$; interior orbits in this face converge to one of the rest points $(1, 1, 1, y')$ with $0 < y' < 1$, see Figure ?

$(*, 0, 1, *)$ — this is the face containing the isolated equilibrium E2. Most orbits in this face converge to $(0, 0, 1, 1) \in$ P3 or to $(1, 0, 1, 0)$ (which is unstable along the edge $(1, *, 1, 0)$ along which there is a connection to $(1, 1, 1, 0) \in$ P2. The saddle point E2 lies on the separatrix, i.e., the manifold separating the two basins of attraction.

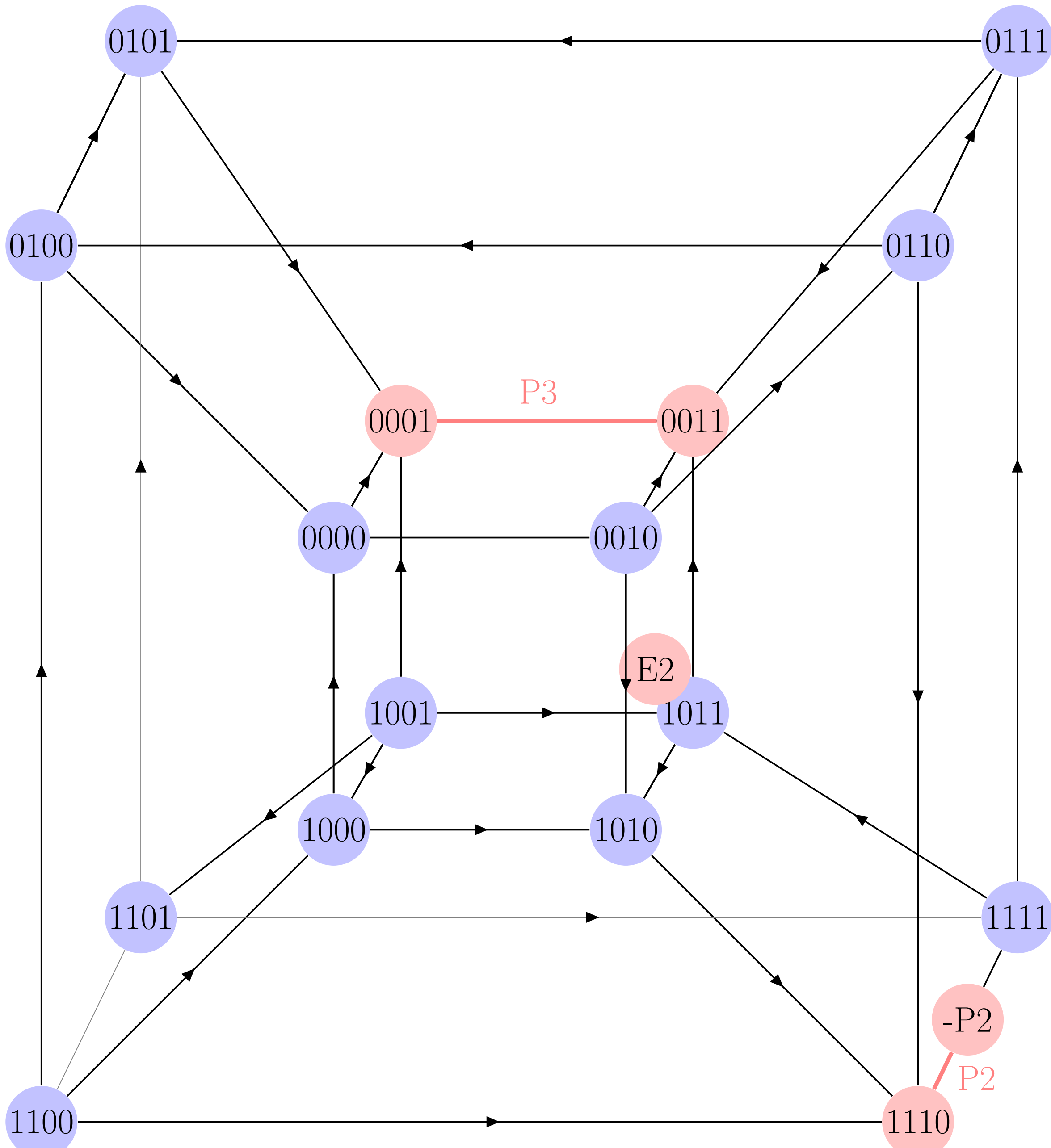
$(*, 0, *, 1)$ — this face contains the edge of restpoints $(0, 0, *, 1)$ which is exactly the equilibrium component P3. Interior orbits in this face converge to one of the NE in P3.

Behavior near P2 and P3

The edge P3 is asymptotically stable under both REP and BR dynamics (all external eigenvalues negative)

Which of the two components P2 and P3 is more stable? In the best response dynamics they are both asymptotically stable. Deciding between the two is a delicate matter of equilibrium selection. For $p \sim \frac{1}{2}$ the component P2 may be more attractive, as $p \uparrow 1$, P3 may become dominant.

However, in the replicator dynamics, only P3 is asymptically stable, whereas P2 is stable and interior attracting (Cressman), but not asymptotically stable, since the whole edge spanned by P2 consists of rest points.


0101
0111
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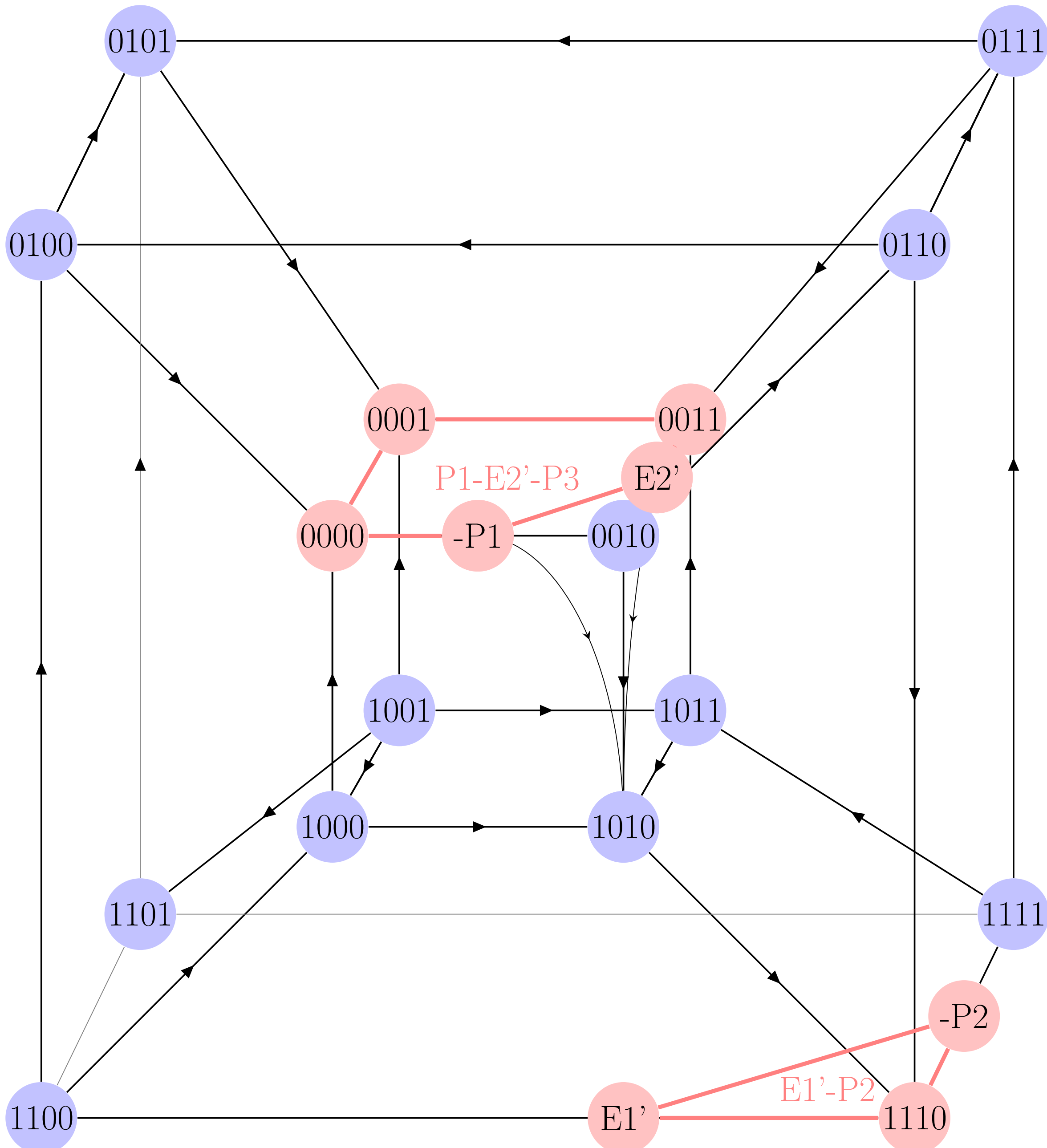
0101
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E2'
P1-E2'-P3
0000
-P1
0010
1001
1011
1000
1010
1101
1111
-P2
E1'-P2
1100
E1'
1110

Phenomena explained:

When prior is low, $p > 1/2$:

- Partially revealing equilibrium (E1):

  costly signal becomes a means to shape the belief of the other; specifically: "push the belief of the other up" $\longrightarrow$ for of "indirect speech"

- (E1) welfare-improving over "no-signaling" equilibrium outcome (P1).

When prior is high, $p > 1/2$:

- both routinely using the costly signal (P2) and routinely not using costly signal (P3) are strategically and evolutionarily stable equilibrium outcomes
  - overstatement (P2) and understatement (P3)
  - P2: Social tragedy: everybody needs to signal, but signal carries no information!
  - P3 can also be interpreted as "countersignaling"
- co-existence of these two equilibrium outcomes $\rightarrow$ possible source of discrimination: when (P2) or (P3) is linked to some other observable characteristic

Equilibrium structure. Class I, case 2: $0 \leq c_1 < c_2 = \mathbf{1}$

- $p < \frac{1}{2}$:

(E*-E1): *perf–part rev* $\quad \mathbf{h} \longrightarrow \mathbf{s} \quad \mathbf{s} \longrightarrow p^{\star} \geq \frac{1}{2} : \mathbf{a}$

Index: $+1$ $\quad \mathbf{l} \longrightarrow \mathbf{s}$ with $< \frac{\mathbf{p}}{\mathbf{1-p}} \quad \bar{\mathbf{s}} \longrightarrow$ low for sure : $\bar{\mathbf{a}}$

(P1): *both use* $\bar{\mathbf{s}}$ $\quad \mathbf{h} \longrightarrow \bar{\mathbf{s}} \quad \mathbf{s} \longrightarrow a$ with $\leq c_1$

Index: 0 $\quad \mathbf{l} \longrightarrow \bar{\mathbf{s}} \quad \bar{\mathbf{s}} \longrightarrow p^{\star} = p < \frac{1}{2} : \bar{\mathbf{a}}$

- $p > \frac{1}{2}$:

(E2) : *partially revealing* $\quad \mathbf{h} \longrightarrow \bar{\mathbf{s}}$ with $\frac{\mathbf{1-p}}{\mathbf{p}} \quad \mathbf{s} \longrightarrow$ high for sure : $\mathbf{a}$

Index: -1 $\quad \mathbf{l} \longrightarrow \bar{\mathbf{s}} \quad \bar{\mathbf{s}} \longrightarrow p^{\star} = \frac{1}{2} : \mathbf{a}$ with $1 - c_1$

(E*-P2) : *perf rev—both* s $\quad \mathbf{h} \longrightarrow \mathbf{s} \quad \mathbf{s} \longrightarrow p^{\star} = p > \frac{1}{2} : \mathbf{a}$

Index: $+1$ $\quad \mathbf{l} \longrightarrow \mathbf{s}$ with any prob $\quad \bar{\mathbf{s}} \longrightarrow$ low for sure : $\bar{\mathbf{a}}$

(P3): *both use* $\bar{s}$ $\quad \mathbf{h} \longrightarrow \bar{\mathbf{s}} \quad \mathbf{s} \longrightarrow \mathbf{a}$ with any prob

Index: $+1$ $\quad \mathbf{l} \longrightarrow \bar{\mathbf{s}} \quad \bar{\mathbf{s}} \longrightarrow p^{\star} = p > \frac{1}{2} : \mathbf{a}$

Equilibrium structure. Class I, case 3: $0 \leq c_1 < \mathbf{1} < \mathbf{c_2}$

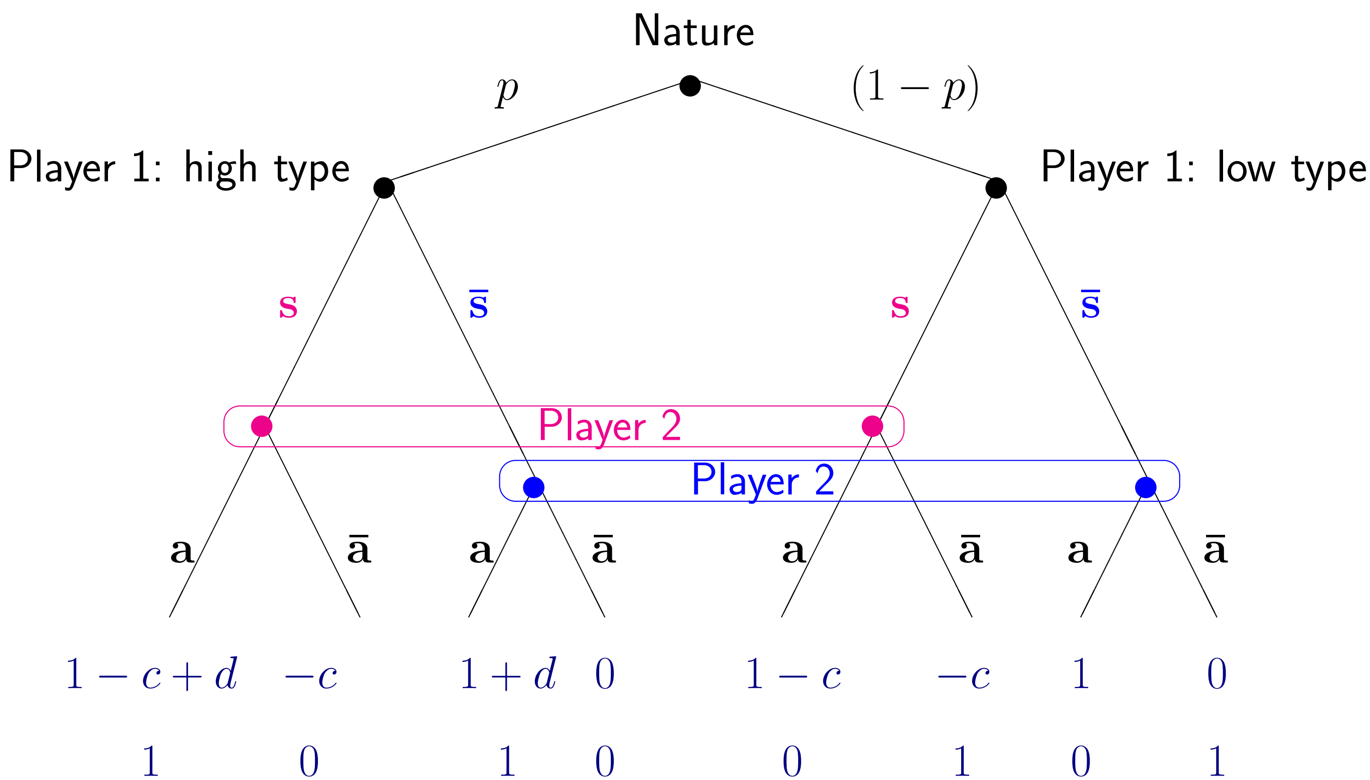
- $p < \frac{1}{2}$:

(E*): *perfectly revealing*	$\mathbf{h} \longrightarrow \mathbf{s}$	$\mathbf{s} \longrightarrow \mathbf{a}$
Index: $+1$	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow \bar{\mathbf{a}}$
(P1) : *both use* $\bar{s}$	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$	$\mathbf{s} \longrightarrow \mathbf{a}$ with $\leq \mathbf{c_1}$
Index: 0	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow \bar{\mathbf{a}}$

- $p > \frac{1}{2}$:

(E2): *partially revealing*	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$ with $\frac{\mathbf{1-p}}{\mathbf{p}}$	$\mathbf{s} \longrightarrow \mathbf{a}$
Index: -1	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow \mathbf{a}$ with $\mathbf{1} - \mathbf{c_1}$
(E*) : *perfectly revealing*	$\mathbf{h} \longrightarrow \mathbf{s}$	$\mathbf{s} \longrightarrow \mathbf{a}$
Index: $+1$	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow \bar{\mathbf{a}}$
(P3) : *both use* $\bar{s}$	$\mathbf{h} \longrightarrow \bar{\mathbf{s}}$	$\mathbf{s} \longrightarrow \mathbf{a}$ with any prob.
Index: $+1$	$\mathbf{l} \longrightarrow \bar{\mathbf{s}}$	$\bar{\mathbf{s}} \longrightarrow \mathbf{a}$

Class II: uniform costs, differential gains: Handicap Principle



	$\mathbf{aa}$	$\mathbf{a\bar{a}}$	$\mathbf{\bar{a}a}$	$\mathbf{\bar{a}\bar{a}}$
$\mathbf{ss}$	$1-c+pd,\ p$	$1-c+pd,\ p$	$-c,\ 1-p$	$-c,\ 1-p$
$\mathbf{s\bar{s}}$	$1-pc+pd,\ p$	$p(1-c)+pd,\ 1$	$-pc+(1-p),\ 0$	$-pc,\ 1-p$
$\mathbf{\bar{s}s}$	$1-(1-p)c+pd,\ p$	$(1-p)(1-c),\ 0$	$p-(1-p)c+pd,\ 1$	$-(1-p)c,\ 1-p$
$\mathbf{\bar{s}\bar{s}}$	$1+pd,\ p$	$0,\ 1-p$	$1+pd,\ p$	$0,\ 1-p$

Class II

Same equilibrium structure as class I:

$$\text{replace } c_1 \text{ by } \frac{c}{1+d}$$

Combination of class I and II:

$$\text{replace } c_1 \text{ by } \frac{c_1}{1+d}$$

To close:

outlook to applications in the stuy fo language:

accents

THOSE OF US who move from the provinces pay a toll at the city's gate, a toll that is doubled in the years that follow as we try to find a balance between what was so briskly discarded and what was so carefully, hesitantly, slyly put in its place. [...] Did I know, they asked, that my accent and tone, indeed my entire body language, had changed when I met their maid? I was almost a different person. Was I aware that I had, in turn, changed back to the person they had met in Egypt once I was alone with them again ? I asked them, did they not speak in different ways to different people? No, they insisted, they did not. Never! They looked at me as if I was the soul of inauthenticity. And then I realized that those of us who move from the periphery to the center turn our dial to different wavelengths depending on where we are and who else is in the room.

(Colm Tóibín, NYRB, July 13, 2017)

Application to sociolinguistics:

- speaking "standard" as a costly signal
- the cost of which is unevenly distributed

  $\rightarrow$ a form of indirect discrimination